Self-consistent interaction of neutrals and shocks in the local interstellar medium

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Abstract

The problem of hydrogen neutrals interacting with the heliospheric bow shock region has received considerable attention recently, motivated primarily by the hope that the Voyager spacecraft may soon encounter the first of the heliospheric boundaries. The complexity of the charge exchange interactions has limited our analytic understanding so far. In this work we develop a semi-analytic one-dimensional model based upon a double expansion technique to investigate the self-consistent interaction of shock with neutrals. The underlying method uses the Boltzmann transport equation that describes neutral transport, and is coupled to plasma protons predominantly through charge exchange processes. The gyrotropic distribution function of the neutrals is first expanded in terms of Legendre polynomials followed by associated Laguerre harmonics. The resulting set of equations can be cast into a square matrix whose eigenvalues depend upon sources and the plasma distribution. The interstellar protons, in a high β limit, are described by a hydrodynamic fluid with source terms that couple neutrals and plasma and further modify both distributions in a self-consistent manner.

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I. INTRODUCTION

The interaction of the supersonic solar wind with the local interstellar medium (LISM) is a problem of great topicality, as well as promising fundamental insights for stellar systems interacting with the interstellar envirnoment in general. The LISM is essentially a partially ionized gas, and the coupling of atoms to the solar wind and LISM plasma greatly complicates the interaction [1]. Such interactions have been been investigated using numerical models such as a multifluid description where plasma and neutrals are treated hydrodynamically [2], or by treating neutrals kinetically and the plasma as an unmagnetized hydrofluid [3, 4]. The fluid description of neutrals in the outer heliosphere can however be criticized because their mean free paths are typically comparable to the size of the heliosphere. The virtually collisionless neutrals therefore cannot equilibrate thermally on heliospheric lengthscales which essentially prevents the use of a single fluid neutral description.

The modeling the solar wind-LISM interaction has been based almost exclusively as coupled multi-dimensional models which include neutrals more-or-less self-consistently. By contrast, very little analytical work on the plasma-neutral coupling has accompanied the large-scale modeling, and this is hampering our understanding of the very complicated interaction. Here we develop a simple one-dimensional (1D) transport equation for neutral atoms and investigate self-consistently their influence on the structure and properties of hydrodynamic shocks. Since the neutrals are coupled to the plasma via charge exchange, we might expect the plasma to be cooled and the neutrals to be heated downstream of the shock. However some heated neutrals can propagate back upstream, where they can undergo secondary charge exchange and so heat the upstream unshocked fluid. Clearly one can expect neutral atoms interacting with a shock wave to modify both the structure and possibly the character of a shock. While obviously of interest to the solar wind-LISM interaction, this work will have interesting implications for any shock embedded in a partially ionized medium, ranging from supernova shocks (SNR) to cometary bow shocks.

Beginning with the Boltzmann transport equation for the neutrals, we employ a double expansion method to compute the neutral particle distribution. The production and loss of neutrals are computed similarly and couple to the plasma flow for which a hydrodynamical description is adopted.

II. TRANSPORT OF NEUTRALS

The transport of neutrals across the plasma shock is governed by the time dependent Boltzmann transport equation describing the evolution of the neutral distribution function $f = f_H(\mathbf{x}, \mathbf{v}, t)$ where $\mathbf{x}, \mathbf{v}, t$ denote the position, velocity and time co-ordinates; thus

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = P(\mathbf{x}, \mathbf{v}, t) - L(\mathbf{x}, \mathbf{v}, t) \tag{1}$$

where P and L describe the production and loss of neutrals, mainly due to charge exchange, and are given by [2]

$$P(\mathbf{x}, \mathbf{v}, t) = f_p(\mathbf{x}, \mathbf{v}, t) \int f(\mathbf{x}, \mathbf{v}', t) |\mathbf{v} - \mathbf{v}'| \sigma_{\text{ex}} d^3\mathbf{v}';$$

$$L(\mathbf{x}, \mathbf{v}, t) = f(\mathbf{x}, \mathbf{v}, t) \int f(\mathbf{x}, \mathbf{v}', t) |\mathbf{v} - \mathbf{v}'| \sigma_{\text{ex}} d^3\mathbf{v}'.$$

Here f_p is the plasma distribution function, $\mathbf{v}'$ is the speed of neutrals and σ_{ex} is the charge exchange (between neutrals and plasma protons) cross-section. $\mathbf{F}$ represents forces experienced by the neutrals due to gravity and raditation pressure. Let us now assume that the neutral particles are gyrotropic and stationary in a frame of the moving plasma fluid. The neutral speed can then be decomposed according to $\mathbf{v} = \mathbf{v}' + \mathbf{u}$, where $\mathbf{u} = u_p$ is bulk flow velocity of the plasma. Upon using spherical co-ordinates, the neutral velocity can be transformed as $\mathbf{v} = v \sin\theta \cos\phi \hat{e}_x + v \sin\theta \sin\phi \hat{e}_y + v \cos\theta \hat{e}_z$. We define $\cos\theta = \mu$. The transport equation describing the evolution of a neutral gyrotropic distribution function along the mean flow direction is given by

$$\begin{aligned}
&\frac{\partial f}{\partial t} + \left(\mathbf{u} \cdot \nabla + \mu v \frac{\partial}{\partial z}\right) f + \left[\frac{\mu F_z}{mv} - \frac{\mu}{v}\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) u_z - \frac{1-\mu^2}{2} \nabla \cdot \mathbf{u} \right. \\
&\left. + \frac{1-3\mu^2}{2} \frac{\partial u_z}{\partial z}\right] v \frac{\partial f}{\partial v} + \frac{1-\mu^2}{2} \left[\frac{2F_z}{mv} - \frac{2}{v}\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) u_z + \mu \nabla \cdot \mathbf{u} - 3\mu \frac{\partial u_z}{\partial z}\right] \frac{\partial f}{\partial \mu} \\
&= P(\mathbf{x}, \mathbf{v}, t) - L(\mathbf{x}, \mathbf{v}, t),
\end{aligned} \tag{2}$$

where u_z is the $\hat{z}$ component of the plasma velocity. The neutral transport Boltzmann Eq. (2) is rather complicated by itself and is coupled to the plasma evolution in two ways. Firstly, the spatial and temporal evolution of plasma velocity modifies the neutral distribution. Secondly, the sources, i.e. the production and the loss of neutrals by charge exchange, depend upon the relative speed of the neutrals and the solar wind plasma proton species. The absolute magnitude of this relative speed can be expressed in terms of Legendre polynomials as $|\mathbf{v} - \mathbf{v}'| = \sum_{n=0}^{\infty} a_n(v, v') P_n(\cos\xi)$, where the coefficients a_n are to be determined

by orthogonality. Using the addition theorem of spherical harmonics for the orthogonal Legendre polynomials and carrying out the angular integration between 0 and 2π allows the production and loss terms for the neutral species to be written as

$$P(\mathbf{x}, \mathbf{v}, t) = 2\pi f_p(v) \int_0^\infty \int_{-1}^{+1} v'^2 f(v') \sigma_{\rm ex} \sum_{n=0}^{\infty} a_n(v, v') P_n(\mu) P_n(\mu') d\mu' dv' \tag{3}$$

$$L(\mathbf{x}, \mathbf{v}, t) = 2\pi f(v) \int_0^\infty \int_{-1}^{+1} v'^2 f_p(v') \sigma_{\rm ex} \sum_{n=0}^{\infty} a_n(v, v') P_n(\mu) P_n(\mu') d\mu' dv' \tag{4}$$

The charge exchange parameter $\sigma_{\rm ex}$ has a logarithmically weak dependence on the relative speed of the neutrals and plasma. It is therefore assumed to be constant throughout our analysis.

III. SOLUTION BY EXPANSION

The multidimensionality of the neutral distribution function, described by Eq. (2), poses severe difficulties in its analytic solution. We are therefore primarily concerned with reducing the dimension of the Eq. (2). For this purpose, the analytic approach developed by Zank et al [5] has been used to reduce, first, the μ dependence of the neutral particle distribution function. The distribution function is expanded in terms of Legendre polynomials as $f(z, \mu, v, t) = \sum_{n=0}^{\infty} (2n+1) P_n(\mu) f_n(z, v, t)$. With the help of the orthogonality condition on the Legendre polynomials $\int_{-1}^{+1} P_n(x) P_m(x) = \delta_{m,n}/(2n+1)$, where $\delta_{m,n}$ is the Dirac delta function equals unity for $m = n$ and zero otherwise, the nth component of the evolution of the neutral distribution function $f_n(z, v, t)$ is

$$\begin{aligned}
&\frac{\partial}{\partial t} f_n + u_z \frac{\partial}{\partial z} f_n + \left(\frac{n+1}{2n+1}\right) v \frac{\partial}{\partial z} f_{n+1} + \frac{nv}{2n+1} \frac{\partial}{\partial z} f_{n-1} + \frac{F_z}{m} \frac{1}{2n+1} \left[n \frac{\partial}{\partial v} f_{n-1} \right. \\
&\quad \left. + (n+1) \frac{\partial}{\partial v} f_{n+1} \right] - \frac{v}{2n+1} \left(\frac{u_z}{v} + 1 \right) \frac{\partial u_z}{\partial z} \left\{ \frac{n(n-1)}{2n-1} \frac{\partial}{\partial v} f_{n-2} + \left[\frac{(n+1)^2}{2n+3} + \right. \right. \\
&\quad \left. \left. \frac{n^2}{2n-1} \right] \frac{\partial}{\partial v} f_n + \frac{(n+1)(n+2)}{2n+3} \frac{\partial}{\partial v} f_{n+2} \right\} + \frac{1}{2n+1} \left[(n+1)(n+2) f_{n+1} \right. \\
&\quad \left. - n(n-1) f_{n-1} \right] \left(\frac{F_z}{mv} + \frac{u_z}{v} \frac{\partial u_z}{\partial z} \right) - \frac{\partial u_z}{\partial t} \left[n \frac{\partial}{\partial v} f_{n-1} + (n+1) \frac{\partial}{\partial v} f_{n+1} + \right. \\
&\left. \frac{(n+1)(n+2)}{v} f_{n+1} - \frac{n(n-1)}{v} f_{n-1} \right] - \frac{1}{2n+1} \left[n(n+1) \left(\frac{n}{2n-1} - \frac{n+1}{2n+3} \right) f_n \right. \\
&\quad \left. + \frac{(n+1)(n+2)(n+3)}{2n+3} f_{n+2} - \frac{n(n-1)(n-2)}{2n-1} f_{n-2} \right] \frac{\partial u_z}{\partial z} = \frac{4N}{M\sqrt{\pi}} \exp\left(-v^2\right) \\
&\quad \times \int_0^\infty dv' v'^2 \frac{a_n}{2n+1} f_n(v') - \frac{4N}{M\sqrt{\pi}} f_n(v) \int_0^\infty dv' v'^2 \exp\left(-v'^2\right) \frac{a_n}{2n+1},
\end{aligned} \tag{5}$$

where $N = (n_{p_0}/n_{h_0})^{1/2}$ is a dimensionless parameter and depends upon the ratio of equilibrium plasma (n_{p_0}) and neutral (n_{h_0}) densities, $M = u_{z_0}/v_{th}$ is plasma Mach number with u_{z_0} as upstream shock speed and v_{th} is thermal velocity of plasma protons. In the non-dimensional Eq. (5), we normalize the distribution function, length-scales, time and velocity of neutrals respectively as $\bar{f} = f/n_{p0}\pi^{-3/2}/v_{th}^3$, $\bar{z} = z\sigma_{\rm ex}(n_{p_0}n_{h_0})^{1/2}$, $\bar{t} = t/v_{th}\sigma_{\rm ex}(n_{p_0}n_{h_0})^{1/2}$, $v = v'/v_{th}$. We have neglected the force terms assuming that gravity and radiation pressure balance each other. The rhs of Eq. (5) represents source terms due to constant charge exchange process and couples the plasma to the neutral dynamics. To work with this equation, the leading order harmonics, i.e. $f_0, f_1, f_2, \cdots$, are good enough to describe a nearly complete distribution of the neutral particles as they contribute predominantly to the entire distribution function. Since the neutral distribution function still depends upon position and velocity of the particles which may vary in time, we choose another expansion to eliminate the neutral velocity using $f_m(z, v, t) = \sum_{m=0}^{\infty} \Theta_m(z, t)e^{-v}L_m^{\mu}(v)$ which doesn't lead to unphysically growing solution due to an exponentially damped weighting function in v. This results in a set of one-dimensional partial differential equations for $\Theta_m(z, t)$ which can be cast into a square matrix form of $n \times m$ order. The order of the matrix depends mainly upon how many harmonics are considered in the two expansions. The general form of the matrix equations, however, looks like

$$\left(\frac{\partial}{\partial t} + u_z\frac{\partial}{\partial z}\right)\bar{\Lambda} + \bar{\Theta}\bar{\Lambda} = 0 \tag{6}$$

$$\text{with } \bar{\Lambda} = \begin{bmatrix} \phi_0 \\ \psi_0 \\ \xi_0 \\ \phi_1 \\ \psi_1 \\ \xi_1 \\ \phi_2 \\ \psi_2 \\ \xi_2 \\ \vdots \end{bmatrix}, \quad \bar{\Theta} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & 0 & \dots & d_1/M \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & 0 & \dots & d_2/M \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & 0 & \dots & d_3/M \\ 0 & 0 & 0 & c_{44} & c_{45} & c_{46} & c_{47} & \dots & d_4/M \\ 0 & 0 & 0 & c_{54} & c_{55} & c_{56} & c_{57} & \dots & d_5/M \\ 0 & 0 & 0 & c_{64} & c_{65} & c_{66} & c_{67} & \dots & d_6/M \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{77} & \dots & d_7/M \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & c_{87} & \dots & \vdots \end{bmatrix}$$

where ∂_z is partial derivative w.r.t the z co-ordinate. The coefficients (C's, and d's, whose actual expression are not shown here) of the coupling matrix could in principle be a complicated function of sources, the spatial and temporal evolution of the plasma species and so too are the eigenvalues. Here $\phi_n, \psi_n, \xi_n, \cdots$ are related to $f_1, f_2, f_3, \cdots$ by means of the Laguerre polynomial expansion. The boundary condition (BC), which is crucial in this problem, can be set appropriately using the two expansions. For a typical problem, we inject Maxwellian neutrals, thermally equilibrated with respect to the plasma, through the left boundary, while a Dirichlet type-BC is set for the right boundary in the simulation. The entire system of equations can be integrated numerically in the presence of a high-β (β being the ratio of pressure and magnetic energy of plasma particles) supersonic plasma for which we adopt a hydrodynamical description, as discussed in the subsequent section.

IV. PLASMA DYNAMICS

The plasma protons can be described by the usual compressible gasdynamical equations in which neutrals can influence the evolution of plasma proton in a self-consistent manner.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{7}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) + \nabla p = \mathbf{Q}_m(\mathbf{x}, t), \tag{8}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1}\right) + \nabla \cdot \left(\frac{1}{2}\rho u^2 \mathbf{u} + \frac{\gamma}{\gamma - 1} p \mathbf{u}\right) = Q_e(\mathbf{x}, t). \tag{9}$$

While the plasma proton density is conserved, i.e. Eq. (7), the momentum and the energy, Eqs. (8) & (9), can be changed by the sources Q_m and Q_e respectively. The sources further couple the plasma fluid with the neutral flow through the charge exchange processes. These can be described by transfer integrals as computed below [3],

$$\mathbf{Q}_m(\mathbf{x}, t) = \int\int (\mathbf{v}_H - \mathbf{v}_p)|\mathbf{v}_H - \mathbf{v}_p| \sigma_{\mathrm{ex}} f_H(\mathbf{x}, \mathbf{v}_H, t) f_p(\mathbf{x}, \mathbf{v}_p, t) d^3\mathbf{v}_H d^3\mathbf{v}_p,$$

$$Q_e(\mathbf{x}, t) = \frac{1}{2}\int\int (v_H^2 - v_p^2)|\mathbf{v}_H - \mathbf{v}_p| \sigma_{\mathrm{ex}} f_H(\mathbf{x}, \mathbf{v}_H, t) f_p(\mathbf{x}, \mathbf{v}_p, t) d^3\mathbf{v}_H d^3\mathbf{v}_p.$$

For a Maxwellian one-dimension plasma distribution, the above integrals reduce to the normalized form

$$Q_m(z, t) = \frac{N}{M\pi^{3/2}} \int_{v'} \left(\frac{\sqrt{\pi}}{2} v'^2 - v_{th} v' + \frac{\sqrt{\pi}}{4} v_{th}^2\right) f(v') dv', \tag{10}$$

$$Q_e(z,t) = \frac{N}{M\pi^{3/2}} \int_{v'} \left[\left(\frac{v'^3}{2} - \frac{v'}{4} v_{th}^2 \right) \sqrt{\pi} v_{th} - \frac{v_{th}^2}{2} \left(v'^2 + v_{th}^2 \right) \right] f(v') dv', \tag{11}$$

where all the symbols have their usual meanings. Equations (7) to (11), describing the evolution of the plasma protons, along with the nth order matrix equation (for the neutrals), to form dynamically a self-consistent problem.

V. CONCLUSION

We have developed a self-consistent one-dimensional semi-analytic model for transport of the neutrals governed by the time dependent Boltzmann equation. The coupling of the neutrals to plasma protons, in a high β collisionless interstellar flow, occurs predominantly through the neutral-proton charge exchange. The latter influences both the neutral and the plasma dynamics significantly. A double-expansion method for the truncated neutral Boltzmann equation was developed to seek a self-consistent solution of the neutral-plasma coupled system. The numerical simulation of the entire system of equations is underway and will be presented elsewhere.

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[1] Zank, G. P., Sp. Sci. Rev., **89**, 413-688, (1999).

[2] Pauls, H. L., Zank, G. P., and Williams, L. L., J. Geophys. Res. **100**, 21,595-21,604 (1995).

[3] Mueller, H. -R., Zank, G. P., and Lipatov, A. S., J.G.R. 105, 27419, (2000).

[4] Baranov, V. B., and Malama, Y. G., J. Geophys. Res. **98**, 15,157 (1993).

[5] Zank, G. P., Lu, J. Y., Rice, W. K. M., and Webb, G. M., J. Plasma Phys. **64**, 507-541 (2000).